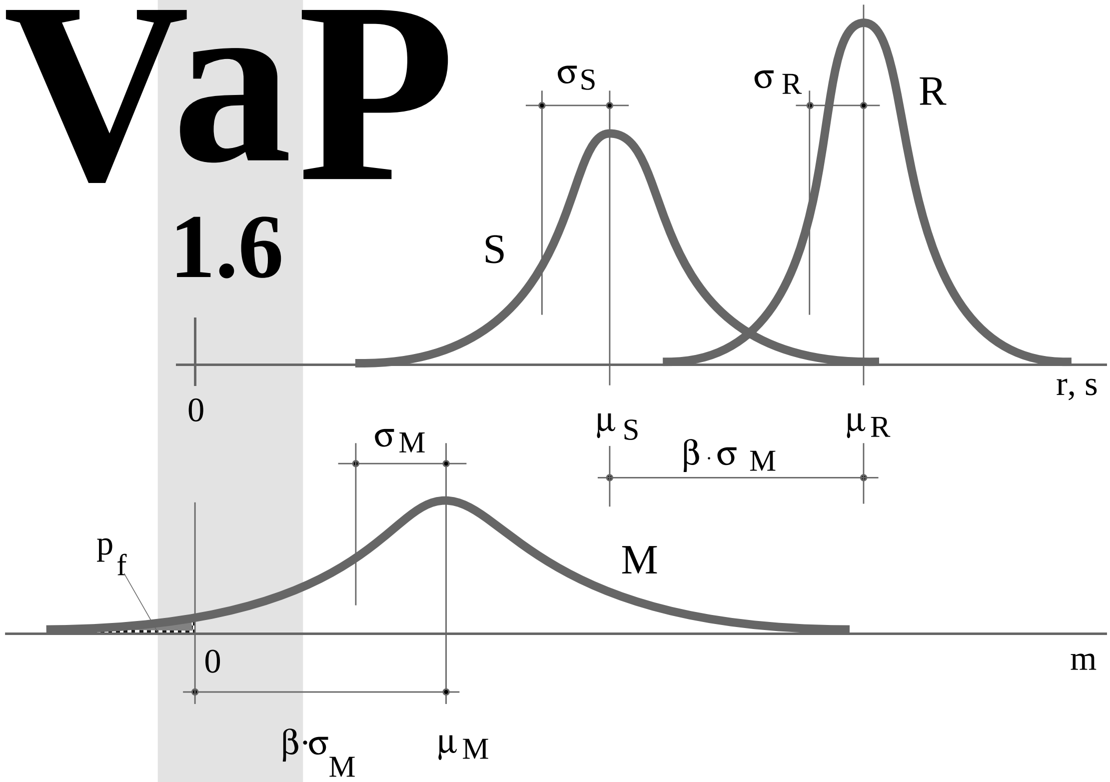
VaP
1.6



for Windows™

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Function and Purpose of *VaP*

The **Va**riables **P**rocessor (***VaP***) enables the user of the program to deal with stochastic quantities, so-called variables, in some given mathematical expression. In view of one of the applications of the program, this expression is called a limit state function (LSF). The program lends itself to reliability analysis, but may be used in a much wider context when evaluating the influence of variables for problems encountered in other fields of engineering practice.

At first, the limit state function G(**X**) representing the problem at hand is defined using the usual mathematical notation and concrete terms for the basic variables **X**. The variables then have to be described by choosing among a set of several distribution types. The results can be produced as a probability of failure, as the first four moments, or as a histogram of the resulting stochastic quantity G.

Requirements, Installation and Starting *VaP*

The program *VaP*, optimised for 16 bit processors, uses the graphical interface Windows™. The PC must be at least of the 386-family. *VaP* needs less than 2.5 MB of hard disk space. When using the Monte Carlo method, faster processors are obviously of advantage.

To install *VaP* on your computer simply insert the *VaP* installation disk, choose the *Run* command from the menu *File* and type **a:\install**.

A double click on the *VaP* icon starts the program and the main VaP window appears as a standard Windows™ application with its typical user interface and menu structure.

Menus

The menu bar provides the essential commands and functions for processing stochastic quantities.

File	
New	Create a new *VaP* document
Open...	Open an existing *VaP* document
Close	Close the active *VaP* document
Save	Save the active *VaP* document
Save As...	Save under a new name
Page Setup	Setup the page
Printer Setup	Setup the printer
Print...	Print the active *VaP* document
Exit	Quit the program

All definitions of limit state functions and basic variables are introduced and may be changed at any time during runtime of the program. The limit state function G(**X**) is written as an algebraic expression and is checked automatically for correct syntax and consistency. Also, all variables are checked to ensure proper definition.

Define	
Function	Define a LSF, open the *Inspector* panel
Variables	Define Variables belonging to a LSF
DeleteLSF	Delete the active LSF
ReportLSF	Report the active LSF and its variables
Undo	Undo the previous delete action

Different methods of analysis are implemented in *VaP*. *VaP* calculates the moments $E[G(\mathbf{X})^n]$ following procedures proposed by Evans, the probability of failure $p_f = P[G(\mathbf{X}) \leq 0]$ using the well known FORM procedures and is able to produce a histogram of G(X) based on Crude Monte Carlo techniques.

Method	
G(E(X))	Evaluation with the expected values
FORM	First Order Reliability Method
MC...	Crude Monte Carlo Method
Moments	Numerical Integration Method
Expectation	Expectation

VaP offers the possibility to define a number of LSFs and basic variables. All defined limit states are stored in the *Workspace* and are accessible from the menu *Activate*. The maximum number of LSFs is limited to a number of 30.

Activate	
G	LSF G
G1	LSF G1
G2	...

Definition of a Limit State Function

After starting the program at first a new document must be created (Choose *New* from the menu *File* or use the toolbar). An empty *Report* window with simple text editing features appears. Choose *Function* from the menu *Define* to see the *Inspector* panel. The algebraic expression of the limit state function can be typed in the upper sub-window and is checked, parsed, and compiled by pressing the *Enter* button. If there are no syntactical errors in the expression the *Inspector* shows, in the lower sub-window, all variables contained in the limit state function.

The program syntax requires a function name, spelled by one or more characters, followed by an equals sign (=) and the algebraic expression. In this expression, a variable is introduced with a series of characters beginning with a letter and distinguishing between capital and small letters. Notice that used function names cannot be selected as variable names, and vice versa.

VaP includes the possibility to define several different LSFs in the same Workspace by renaming the original function in the upper sub-window, (e.g. G to G1) and changing its expression. Switching back and forth between these is done using the menu *Activate*.

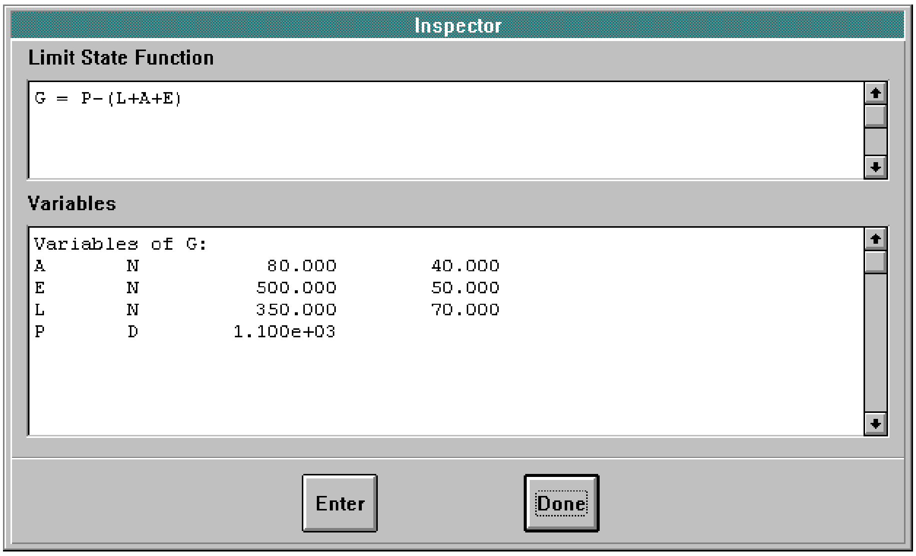


Figure 1: Inspector panel

To define a limit state function, the following operators and functions may be used:

- +,-,*,/
- Exponents with a preceding circumflex ^, for example x^2, x^(a+b) or x^-2, resp. SQRT(...) and SQR(...) as alternatives for writing x^(1/2) and x^2
- Transcendental functions: COS(...), SIN(...), TAN(...), ARCCOS(...), ARCSIN(...), ARCTAN(...), COSH(...), SINH(...), TANH(...), ARCOSH(...), ARSINH(...), ARTANH(...), EXP(...), LN(...)

- Standard normal distribution with probability density function PDFN(...), cumulative distribution function CDFN(...), and inverse of cumulative function INVN(...). For example G = CDFN(0.5)

- ABS(...), as well as the functions MIN(... , ...) and MAX(... , ...), which must be separated by commas as they require two or more arguments

The arguments of trigonometrical functions and the results of their inverse functions have degrees as the unit. All above-mentioned abbreviations, the constants PI, the factor DEG (describing the ratio between degrees and radians) and the abbreviations of the LSFs, e.g. G1, are reserved names and may not be used for defining variables.

The expectation $E[p_f]$ of the probability of failure, the so-called total probability, may be calculated by integration of conditional probabilities $p_f(\mathbf{x})$ over the probability densities of the variables **X**. To solve this problem, a bar sign (|) has to be set at the end of the expression, e.g. G = A-B*C|B. Then, all basic variables to be integrated follow, each separated by a blank space. An example for the use of this expectation operator is shown in [Petschacher, 1993].

Definition of Variables

Now a distribution type must be assigned to each of the variables. Each variable is defined globally inside a *Workspace*. By choosing *Variables* from the *Define* menu, the *Variables* panel appears. Each variable, in turn, is assigned a distribution type with corresponding parameters. Alternatively, the modus *Moments*, with expectation, standard deviation, and, if necessary, lower and upper bounds, or the modus *Parameters* with parameters specific to the distribution type may be chosen. The modus *Moments* is set by default.

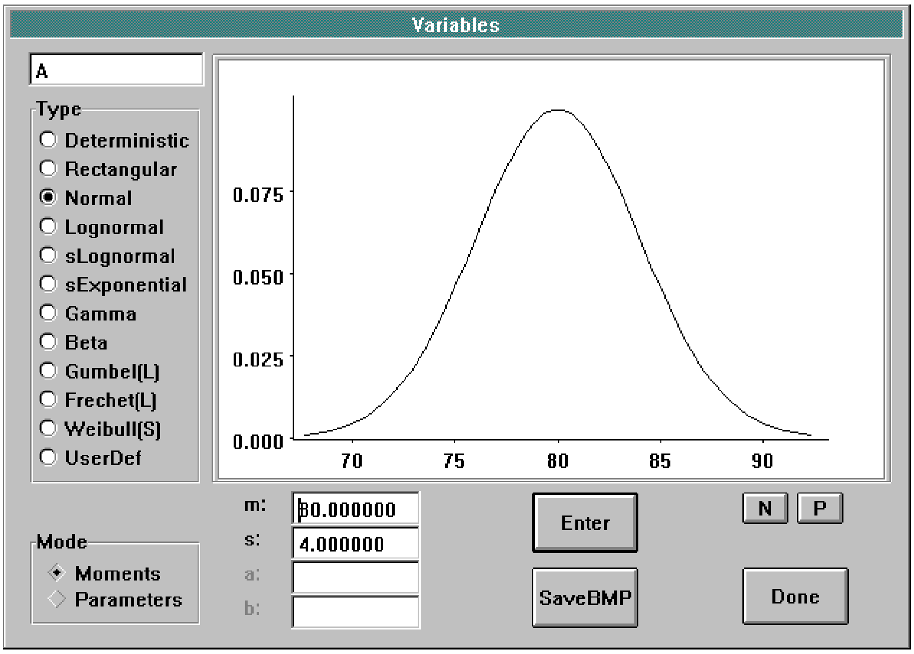


Figure 2: Variables panel

The definition of the variable is read into the program by pressing the *Enter* button. If all corresponding parameters fulfil the conditions given by the distribution type, the probability density function (pdf) of the basic variable appears as a graphical screen output. The pdf may be saved as a bitmap file (**.bmp*) by pressing the *SaveBMP* button.

By using the *N* button the next variable in alphabetical order appears, with the *P* button the previous one.

Variables may also be defined as histograms. By selecting *UserDef* in the *Variables* window and by clicking the *Enter* button a *UserDef* panel appears. The lower and the upper bounds must be defined, followed by a sequence of frequencies separated by pressing the space bar on the keyboard. By clicking *Enter*, the program will automatically calculate the class width. It is also possible to open previously saved histograms with the *Open* button (**.his*). Histograms may be saved by clicking *Save*. Upon clicking *Done*, the histogram appears in the *Variables* panel.

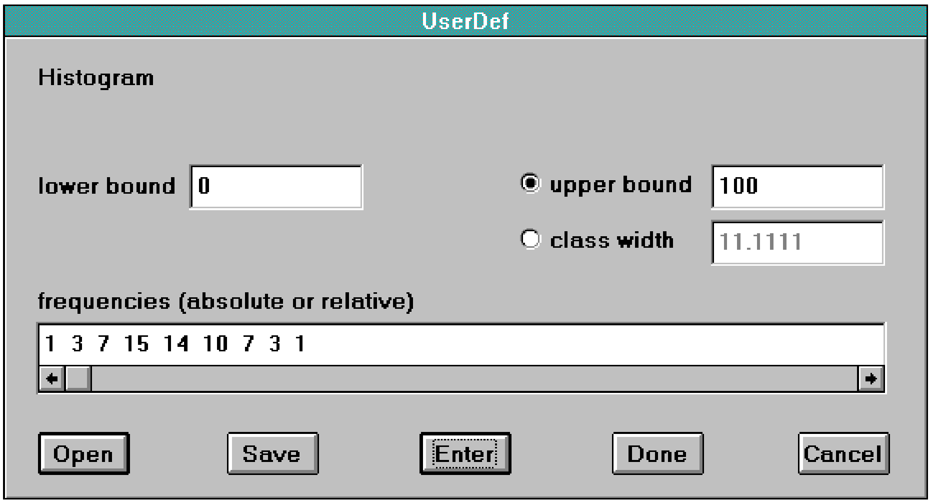


Figure 3: Entering a histogram

The *Done* button ends the definition procedure for the variables. It is good practice to save all the entries under a suitable name using the *Save* button under *File*.

There is no possibility of defining and handling correlations between basic variables.

For more information about the definition of variables, see *Definition of Distribution Parameters*.

Analyses of a Limit State Function

Choosing *G(E[X])* from the menu *Methods* calculates the value of G using the expectations of the variables.

An analysis based on the First Order Reliability Method (FORM) [Rackwitz, 1977] is performed by choosing *FORM* from the menu *Method*. The results, the so-called Hasofer/Lind index (also known as the geometrical β-value), the sensitivity factors α and the design values of the variables are printed to the *Report* window. *FORM* sometimes has difficulties with user defined variables, due to the particular shape of the corresponding histograms.

By choosing *MC...* from the menu *Method* a panel for setting all elements for a Crude Monte Carlo [Rubinstein, 1981] analysis appears on the screen. The *Option Automatic* is set by default enabling the user, by pressing *Go!*, to start the simulation. *Go!* then changes its name to *STOP!*, so that pressing it a second time interrupts the simulation. The results displayed in the window are the first four moments, the probability of failure and a graphical representation of the results as a histogram. The graphic can be saved as a bitmap file (**.bmp*) by pressing the *SaveBMP* button. The user may choose under *Options Define Values* to change the number of classes, runs and simulations (limited to 10'000'000) and the bounds for the display of the histogram on the screen. Pressing the *Done* button adds the numerical results, including the third and forth moments, to the *Report* window.

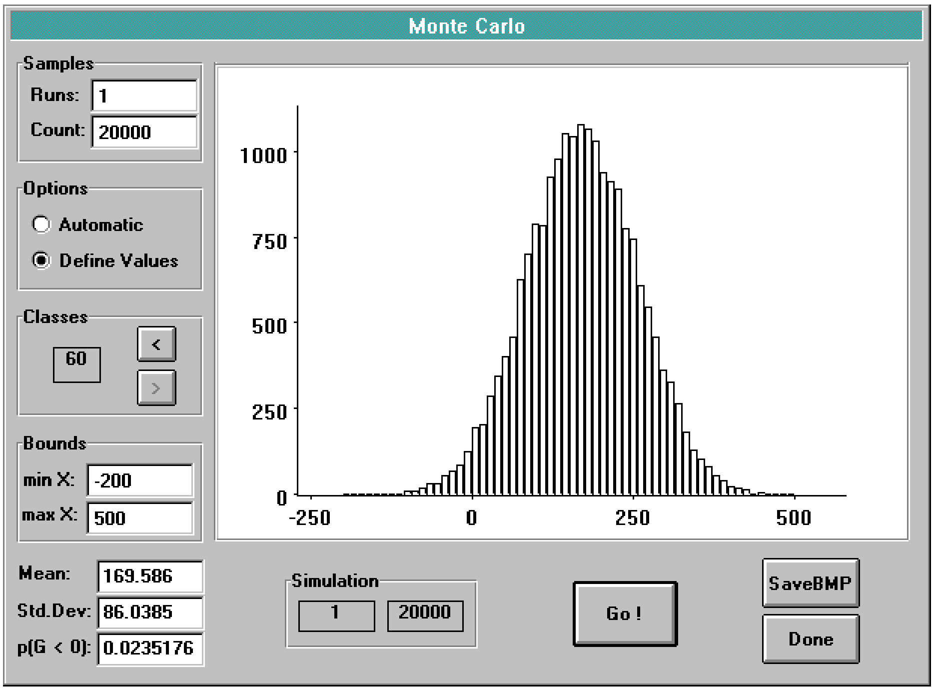


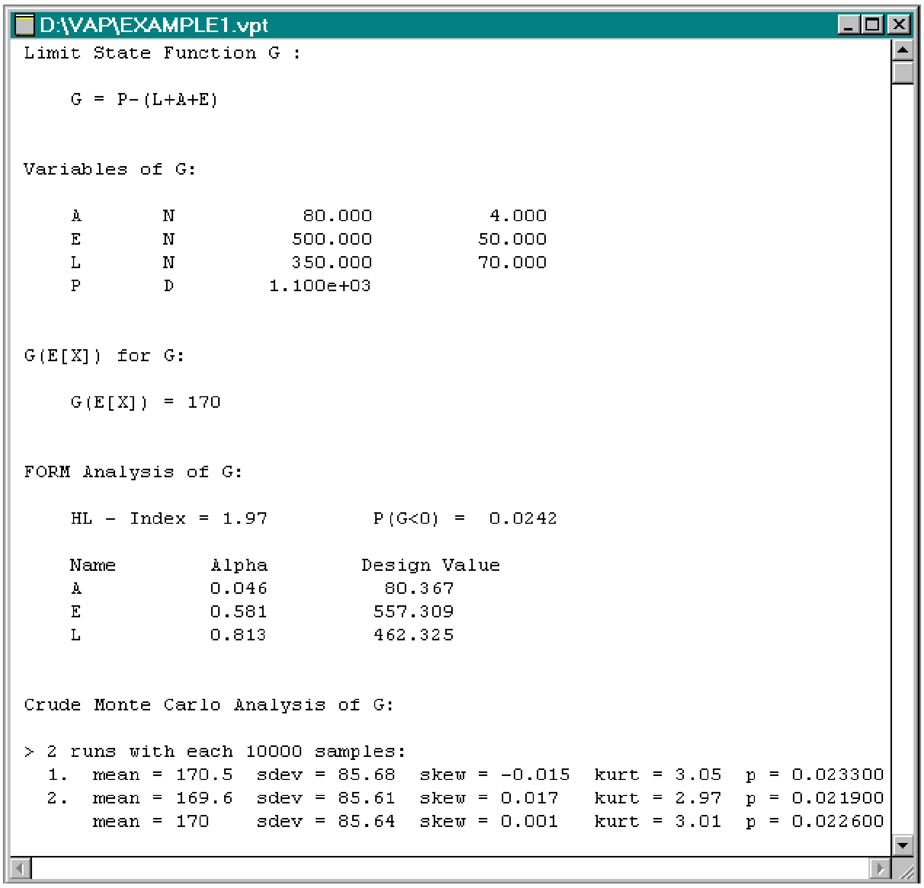
Figure 4: Monte Carlo panel

The calculation of the first four moments using numerical integration procedures [Evans, 1972] can be carried out by choosing *Moments* from the menu *Method*. Additionally *VaP* calculates the parameters of the respective Johnson curve, and an approximation for the probability of failure p_f. For more information about this curve see [Petschacher, 1993; Hill, 1976; Draper, 1952]. The parameters g, d, xl, and xi shown, correspond to γ, δ, λ and ξ in [Petschacher, 1993].

The expectation E(G(X)) of the function is calculated choosing *Expectation* from menu *Methods*.

Editing the Report

In order to relate results to the respective input data, it is recommended to write a report of all the input data to the *Report* window whenever definitions of variables or LSFs are changed. For this purpose, choose *ReportLSF* from the menu *Define*. Results can, optionally, be commented by the user, taking advantage of the editing capabilities of the *Report* window.



```
D:\VAP\EXAMPLE1.vpt
Limit State Function G :

    G = P-(L+A+E)


Variables of G:

    A       N          80.000          4.000
    E       N         500.000         50.000
    L       N         350.000         70.000
    P       D       1.100e+03


G(E[X]) for G:

    G(E[X]) = 170


FORM Analysis of G:

    HL - Index = 1.97          P(G<0) =  0.0242

    Name         Alpha         Design Value
    A            0.046          80.367
    E            0.581         557.309
    L            0.813         462.325


Crude Monte Carlo Analysis of G:

> 2 runs with each 10000 samples:
  1.   mean = 170.5   sdev = 85.68   skew = -0.015   kurt = 3.05   p = 0.023300
  2.   mean = 169.6   sdev = 85.61   skew = 0.017    kurt = 2.97   p = 0.021900
       mean = 170     sdev = 85.64   skew = 0.001    kurt = 3.01   p = 0.022600
```

Figure 5: Report window

Saving and Printing

All definitions inside a *Workspace* and the *Report* window may be saved under **.vap*. Automatically, two different files are created. One is the **.vap* file containing the LSFs and the variables. The second file **.vpt* is a text file, containing the *Report* window. This file can be read by any text processing program.

By choosing *Print* from the menu *File* the *Print* panel appears, enabling printing the Report window on any selected printer.

The results from the *Report* window can be integrated into text processing programs using simply *Copy* and *Paste*. Graphical results previously saved as bitmap files may be introduced into the text as well.

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Help

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Definition of Distribution Parameters

Distribution type	Para-meters	Moments
Deterministic	1 : m	
Rectangular $f_X(x) = \frac{1}{b-a}$ $a \le x \le b\,,\; a \ne b$	1 : a 2 : b	$m = \frac{a+b}{2}$ $s = \frac{b-a}{\sqrt{12}}$
Normal $f_X(x) = \frac{1}{s\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{x-m}{s}\right)^2\right)$ $s > 0\,,\; -\infty < x < +\infty$	1 : m 2 : s	
Lognormal $f_X(x) = \frac{1}{\zeta x\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2\right)$ $\zeta > 0,\; 0 < x < \infty$	1 : λ 2 : ζ	$m = \exp\left(\lambda + \frac{\zeta^2}{2}\right)$ $s = \exp\left(\lambda + \frac{\zeta^2}{2}\right) \cdot \sqrt{\exp\left(\zeta^2\right) - 1}$
sLognormal $f_X(x) = \frac{1}{\zeta(x-\varepsilon)\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{\ln(x-\varepsilon) - \lambda}{\zeta}\right)^2\right)$ $\zeta > 0,\; \varepsilon < x < \infty$	1 : λ 2 : ζ 3 : ε	$m = \varepsilon + \exp\left(\lambda + \frac{\zeta^2}{2}\right)$ $s = \exp\left(\lambda + \frac{\zeta^2}{2}\right) \cdot \sqrt{\exp\left(\zeta^2\right) - 1}$
sExponential $f_X(x) = \lambda \exp(-\lambda(x-\varepsilon))$ $\lambda > 0\,,\; \varepsilon \le x < \infty$	1 : ε 2 : λ	$m = \varepsilon + \frac{1}{\lambda}$ $s = \frac{1}{\lambda}$
Gamma $f_X(x) = \frac{b^p}{\Gamma(p)} \exp(-bx) x^{p-1}$ $b > 0,\;\; p > 0\,,\; 0 \le x < \infty$	1 : p 2 : b	$m = \frac{p}{b}$ $s = \frac{\sqrt{p}}{b}$

Beta $f_X(x)=\dfrac{(x-a)^{r-1}\cdot(b-x)^{t-1}}{(b-a)^{r+t-1}\cdot B(r,t)}$ $a\le x\le b$, $a\ne b$, $r,t\ge 1$	1 : a 2 : b 3 : r 4 : t	$m=a+(b-a)\cdot\dfrac{r}{r+t}$ $s=\dfrac{b-a}{r+t}\cdot\sqrt{\dfrac{rt}{r+t+1}}$
Gumbel (Largest) $f_X(x)=\alpha\exp\left(-\alpha(x-u)-\exp\left(-\alpha(x-u)\right)\right)$ $-\infty<x<+\infty,\ \ \alpha>0$	1 : u 2 : α	$m=u+\dfrac{0.577216}{\alpha}$ $s=\dfrac{\pi}{\alpha\sqrt{6}}$
Frechet (Largest) $f_X(x)=\dfrac{k}{u-\varepsilon}\cdot\left(\dfrac{x-\varepsilon}{u-\varepsilon}\right)^{-k-1}\cdot\exp\left(-\left(\dfrac{x-\varepsilon}{u-\varepsilon}\right)^{-k}\right)$ $\varepsilon\le x<+\infty,\ \ u,k>0$	1 : u 2 : k 3 : ε	$m=\varepsilon+(u-\varepsilon)\Gamma\left(1-\dfrac{1}{k}\right)$ $s=(u-\varepsilon)\sqrt{\Gamma\left(1-\dfrac{2}{k}\right)-\Gamma^2\left(1-\dfrac{1}{k}\right)}$
Weibull (Smallest) $f_X(x)=\dfrac{k}{u-\varepsilon}\cdot\left(\dfrac{x-\varepsilon}{u-\varepsilon}\right)^{k-1}\cdot\exp\left(-\left(\dfrac{x-\varepsilon}{u-\varepsilon}\right)^{k}\right)$ $\varepsilon\le x<+\infty,\ \ u,k>0$	1 : u 2 : k 3 : ε	$m=\varepsilon+(u-\varepsilon)\Gamma\left(1+\dfrac{1}{k}\right)$ $s=(u-\varepsilon)\sqrt{\Gamma\left(1+\dfrac{2}{k}\right)-\Gamma^2\left(1+\dfrac{1}{k}\right)}$

VaP includes additional Parameter Restrictions:

m, s > 0,
$1/3 \le m/s \le 10^4$, (Rectangular, Normal, sExp., Gumbel, Frechet)
$1/3 \le m/s \le 10^2$, (Lognormal, sLognormal)

Special Distributions:

A Rayleigh distribution may be defined, using a Weibull distribution (parameter k = 2 and ε = 0). An exponential distribution is obtained by setting parameter k = 1 again using Weibull. Further, if the parameter k is set to 3.25889 or 3.31125 or 3.43938 or 3.60232, four pseudo-symmetrical cases are obtained.

The first four moments of the Frechet distribution exist only if the parameter k is larger than 4.

A Beta distribution can change to a rectangular (uniform) distribution by setting r = 1 and t = 1, or to a triangular distribution by setting r = 1 and t = 2.